Linear optical implementation of a single mode quantum filter and generation of multi-photon polarization entangled state

XuBo Zou, K. Pahlke and W. Mathis
Electromagnetic Theory Group at THT
Department of Electrical Engineering University of Hannover, Germany

Abstract

We propose a scheme to implement a single-mode quantum filter, which selectively eliminates the one-photon state in a quantum state $\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle$. The vacuum state and the two photon state are transmitted without any change. This scheme requires single-photon sources, linear optical elements and photon detectors. Furthermore we demonstrate, how this filter can be used to realize a two-qubit projective measurement and to generate multi-photon polarization entangled states.

PACS number:03.67.-a, 03.65.Ud

The generation of polarization entangled quantum states of individual photons is an important challenge in the field of information processing with quantum optical systems. In particular the robust nature of this kind of quantum states in respect of decoherence effects provides an advantage over other quantum systems. Related experiments opened a wide field of research [1]. Two-photon polarization entangled states can be used as a simple resource of entanglement. An experimental realization of these states was demonstrated with the parametric down-conversion technique in a nonlinear optical crystal [2]. These two-photon polarization entangled states can be used to test quantum nonlocality [1] and to implement quantum information protocols like quantum teleportation [3], quantum dense coding [4] and quantum cryptography [5]. The entanglement of three-photon states and four-photon states was realized experimentally [6]. These states can be used to demonstrate the extreme contradiction between local realism and quantum mechanics [7] in the so-called Greenberger-Horne-Zeilinger-state (GHZ-state) [8].

So far polarization entangled photon states have only been produced randomly, since there is no way to demonstrate their generation without a measurement and destruction of the outgoing quantum state [9]. This is a severe obstacle for further applications of this state to be used as the input state for following experiments. Some quantum protocols like error correction were designed for maximally entangled quantum states without random entanglement [10]. Thus, there is a strong need in entangled photon sources, which generate maximally polarization entangled multi-photon states in a controllable way. Recently, significant progress was achieved by a proposal to implement probabilistic quantum logic gates by the use of linear optical elements and photon detectors [11]. Although the scheme in Ref.[11] contains only linear optical elements, the optical network is complex and major stability and mode matching problems can be expected in their construction. Several less complicated schemes were presented [12, 13, 14] to implement the quantum logic operation with a slightly lower probability of success. But, all these schemes can be used directly to generate two-photon polarization entangled states from single-photon sources with a probability, which is not greater than 1/16. Another simple scheme [15] was proposed to generate two-photon polarization entangled states and entangled N-photon states from single-photon sources. Recently, a single-photon quantum nondemolition device was proposed for signaling the presence of a single photon in a particular input state $\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle$ without destroying it [16]. In Ref.[17] Hofmann et al proposed a scheme to realize a two-qubit projective measurement

$$P_e = |H,H\rangle\langle H,H| + |V,V\rangle\langle V,V| \,. \tag{1}$$

The two-mode quantum states $|H,H\rangle = |H\rangle_1|H\rangle_2 = |H\rangle_1 \otimes |H\rangle_2$ and $|V,V\rangle = |V\rangle_1|V\rangle_2 = |V\rangle_1 \otimes |V\rangle_2$ are constructed from single-mode quantum states by the tensor product $\otimes$. The quantum state of a horizontally (vertically) polarized photon in the mode i is indicated by $|H\rangle_i(|V\rangle_i)$. This measurement (1) projects the input state of two polarized photons into the two-dimensional subspace of equal polarization (both horizontal or both vertical). These proposals [11, 12, 13, 14, 15] show that photon number detection can be used to generate polarization entanglement, which is in general associated with nonlinear effects. In the present paper, we propose a scheme to realize the two-qubit projective measurement (1), which gives compared to the scheme [17] a four times greater probability of the outcome.
This paper is organized as follows. At first we propose a scheme to implement the single-mode quantum filter, which eliminates the one-photon state in a particular input state $\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle$. Then this filter is applied to realize a two-qubit projective measurement (1) and to show, how multi-photon polarization entangled states can be efficiently generated step by step.

In contrast to the scheme [17] our single-mode quantum filter was inspired by quantum teleportation protocols [18], which make our scheme more efficient for the implementation of the projective measurement (1). Figure 1 shows the scheme of this single-mode quantum filter, which eliminates the one-photon state and transmits the vacuum state and the two-photon state of the input state

$$\Psi_{in} = \alpha|0\rangle_1 + \beta|1\rangle_1 + \gamma|2\rangle_1 \,. \tag{2}$$

A similar setup was used to implement the conditional quantum logic gate [14]. But here, a symmetric beam splitter BS_1 is required. By emitting two single photons $|\Psi_{23}\rangle = |1\rangle_2|1\rangle_3$ into this symmetric beam splitter BS_1 an entangled photon channel is generated:

$$\Psi'_{23} = \frac{1}{\sqrt{2}}(|2\rangle_{2'}|0\rangle_{3'} - |0\rangle_{2'}|2\rangle_{3'}) \,. \tag{3}$$

Our idea is to use the field state $|\Psi_{in}\rangle|\Psi'_{23}\rangle$ directly for the quantum filter operation. The output mode $2'$ of the beam splitter BS_1 is one of the input modes of the second symmetric beam splitter BS_2. The mode 1, which is chosen to be filtered out, is the second input mode of the beam splitter BS_2. Thus, the quantum state transforms to

$$\begin{aligned}\Psi_o \;=\;& |1\rangle_{1''}|1\rangle_{2''}(\alpha|0\rangle_{3'} + \gamma|2\rangle_{3'})/2 + |0\rangle_{1''}|2\rangle_{2''}(\alpha|0\rangle_{3'} - \gamma|2\rangle_{3'})/2\sqrt{2} \\ &+ |2\rangle_{1''}|0\rangle_{2''}(\alpha|0\rangle_{3'} - \gamma|2\rangle_{3'})/2\sqrt{2} + \Psi_{other} \,. \end{aligned} \tag{4}$$

The other terms Ψ_{other} of the total quantum state Ψ_o do not contribute to the three events, which we consider by a photon number measurement on the modes $1''$ and $2''$ by the detectors D_1 and D_2. If each detector detects one photon, the state is projected into the (unnormalized) state

$$\Psi_{out} = \alpha|0\rangle + \gamma|2\rangle . \tag{5}$$

If D_1 detects two photons and D_2 does not detect any photon or vice versa, the state is projected into the (unnormalized) state

$$\Psi'_{out} = \alpha|0\rangle - \gamma|2\rangle . \tag{6}$$

In order to generate the quantum state (5), a $\pi/2$-phase shifter is needed to change the sign of the state $|2\rangle$. Then the desired transformation

$$\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle \longrightarrow \alpha|0\rangle + \gamma|2\rangle \tag{7}$$

will be obtained. This transformation demonstrates that the proposed setup, which is shown in Fig.1, definitely implements the quantum state filter. The probability of this outcome is $\frac{|\alpha|^2+|\gamma|^2}{2}$.
In Ref.[16] Kok et al proposed a different quantum nondemolition device, which transmits only the single-photon state. Thus, these schemes are within the constraint of a particular input state in that respect complementary. Quantum state filters of that kind may find a wide range of application, from quantum projective measurements to the generation of multi photon quantum states.

Now we use our quantum filter concept to implement the two-qubit projective measurement (1). We will demonstrate, that the input state

$$\Phi_{in} = c_0|H\rangle_1|H\rangle_2 + c_1|H\rangle_1|V\rangle_2 + c_2|V\rangle_1|H\rangle_2 + c_3|V\rangle_1|V\rangle_2 \tag{8}$$

will be transformed into the (unnormalized) state

$$\Phi_{out} = c_0|H\rangle_1|H\rangle_2 + c_3|V\rangle_1|V\rangle_2 . \tag{9}$$

Figure 2 shows the required experimental setup to realize this projective measurement by two quantum filters, like they are shown in Fig.1. The half-wave plate HWP_{90} rotates the polarization of the mode 1 by $\pi/2$. The state (8) becomes

$$\Phi_1 = c_0|V\rangle_1|H\rangle_2 + c_1|V\rangle_1|V\rangle_2 + c_2|H\rangle_1|H\rangle_2 + c_3|H\rangle_1|V\rangle_2 . \tag{10}$$

Then mode 1 and the mode 2 are forwarded to a polarization beam splitter PBS_1 to transmit the H-polarized photon and to reflect the V-polarized photon. The system state evolves into

$$\Phi_2 = c_0|HV\rangle_{2'} + c_1|V\rangle_{1'}|V\rangle_{2'} + c_2|H\rangle_{1'}|H\rangle_{2'} + c_3|HV\rangle_{1'} . \tag{11}$$

The quantum state representation of two photons in the same mode i is defined as: $|HV\rangle_i = |VH\rangle_i = |H\rangle_i \otimes |V\rangle_i$. Another one half-wave plate (HWP_{45}) rotates the polarization of the mode $1'$ by $\pi/4$. This corresponds to the transformations $|H\rangle \longrightarrow \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|V\rangle \longrightarrow \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$. The quantum state of the system is transformed into

$$\begin{aligned}\Phi_3 \;=\; & c_0|HV\rangle_{2'} + \frac{1}{\sqrt{2}}|H\rangle_3(c_1|V\rangle_{2'} + c_2|H\rangle_{2'}) \\ & + \frac{1}{\sqrt{2}}|V\rangle_3(-c_1|V\rangle_{2'} + c_2|H\rangle_{2'}) + \frac{c_3}{\sqrt{2}}(|2H\rangle_3 - |2V\rangle_3) .\end{aligned} \tag{12}$$

The mode 3 is injected into another polarization beam splitter PBS_2 so that the system state becomes

$$\Phi_4 = c_0|HV\rangle_{2'} + \frac{1}{\sqrt{2}}|H\rangle_4(c_1|V\rangle_{2'} + c_2|H\rangle_{2'}) + \frac{1}{\sqrt{2}}|V\rangle_5(-c_1|V\rangle_{2'} + c_2|H\rangle_{2'}) + \frac{c_3}{\sqrt{2}}(|2H\rangle_4 - |2V\rangle_5)\,. \tag{13}$$

The mode 4 and the mode 5 pass a single-mode quantum filter, like it is shown in Fig.1, to eliminate the one-photon state of the mode 4 and the mode 5. This filtering process transforms the state (13) to the state

$$\Phi_5 = c_0|HV\rangle_{2'} + \frac{c_3}{\sqrt{2}}(|2H\rangle_{4'} - |2V\rangle_{5'})\,. \tag{14}$$

The mode $4'$ and the mode $5'$ are two input modes of the polarization beam splitter PBS_3. That's why the system state transforms to

$$\Phi_6 = c_0|HV\rangle_{2'} + \frac{c_3}{\sqrt{2}}(|2H\rangle_{3'} - |2V\rangle_{3'})\,. \tag{15}$$

The polarization of the mode $3'$ is rotated by $\pi/4$ to get

$$\Phi_7 = c_0|HV\rangle_{2'} + c_3(|HV\rangle_6)\,. \tag{16}$$

And finally the mode 6 and the mode $2'$ pass through a polarization beam splitter PBS_4 in order to obtain the output state (9) of the implemented two-qubit projective measurement (1). The probability of this outcome is $(|c_0|^2 + |c_3|^2)/4$, which is four times greater than what the proposal [17] makes possible.

Now we show how the multi-photon polarization entangled states can be efficiently generated. At first we consider the generation of the two-photon polarization entangled state $(|H\rangle|H\rangle + |V\rangle|V\rangle)/\sqrt{2}$. We assume that the two polarized photons form the initial state $(|H\rangle_1 + |V\rangle_1)/\sqrt{2} \otimes (|H\rangle_2 + |V\rangle_2)/\sqrt{2}$. These two photons are injected into two input ports of the setup, which is shown in Fig.2. Notice that this setup projects the input state into the two-dimensional subspace of the identical horizontal or vertical polarization. After passing through the setup, the initial state is projected into the two-photon polarization entangled state $(|H\rangle|H\rangle + |V\rangle|V\rangle)/\sqrt{2}$. The pobability of this outcome is 1/8. In the following, we consider the generation of a multi-photon polarization GHZ-state. We assume that a $N-1$-photon polarization GHZ-state $(|H\rangle_1 \cdots |H\rangle_{N-1} + |V\rangle_1 \cdots |V\rangle_{N-1})/\sqrt{2}$ and the Nth independent polarization photon are prepared initially in the state $(|H\rangle_N + |V\rangle_N)/\sqrt{2}$. If the $(N-1)$th photon and the Nth polarization photon inject into two input ports of the setup, which is shown in Fig.2., the N-photon polarization GHZ-state $(|H\rangle_1 \cdots |H\rangle_N + |V\rangle_1 \cdots |V\rangle_N)/\sqrt{2}$ will be obtained. The probability of this outcome is 1/8. This demonstrates that N-photon polarization entangled GHZ-states can be generated from single-photon sources step by step. The probability of the outcome of the total process is $\frac{1}{8^{N-1}}$.

In summary, we have proposed a scheme to implement a quantum filter, which eliminates the one-photon state and transmits the vacuum state and the two-photon state in the particular input state $\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle$. We demonstrated that the quantum filter can be applied to implement a two-qubit projective measurement. This two-qubit projective measurement was also proposed by Hofmann et al [17], but our scheme provides a four times bigger probability of outcome.

In the present scheme, the multi-photon polarization entanglement is produced in a controllable way, so that it can be used to generate multi-photon polarization entanglement for some quantum protocols, which are designed for maximally entangled quantum states without random entanglement [10]. In order to compare the efficiency of our scheme to generate the N-photon polarization GHZ-state with the scheme of Knill et al [11] the same requirements on the sources should be posed. In the case of single-photon sources the scheme of Knill et al gives this outcome with the probability $\frac{1}{16^{N-1}}$. Indeed, in the case of an entangled photon source the scheme [11] becomes nearly deterministic, but the main problem is then shifted to the sources.
One of the difficulties of our scheme in respect to an experimental demonstration consists in the requirement on the sensitivity of the detectors. These detectors should be capable to distinguish between one-photon events and two-photon events. Recently, the development of experimental techniques in this field made considerable progress. A photon detector based on visible light photon counter was reported, which can distinguish between a single-photon incidence and a two-photon incidence with a high quantum efficiency, a good time resolution and a low bit-error rate [19]. Another difficulty is the availability of single-photon sources. Several triggered single-photon sources are available, which operate by means of fluorescence from a single molecule [20] or a single quantum dot [21, 22]. They exhibit a very good performance. More recently a deterministic single-photon source for distributed quantum networks was reported [23], which can emit a sequence of single photons on demand from a single three-level atom strongly coupled to a high-finesse optical cavity. However, our scheme needs a synchronized arrival of many single photons into input ports of many beam splitters. An experimental realization will be challenging.

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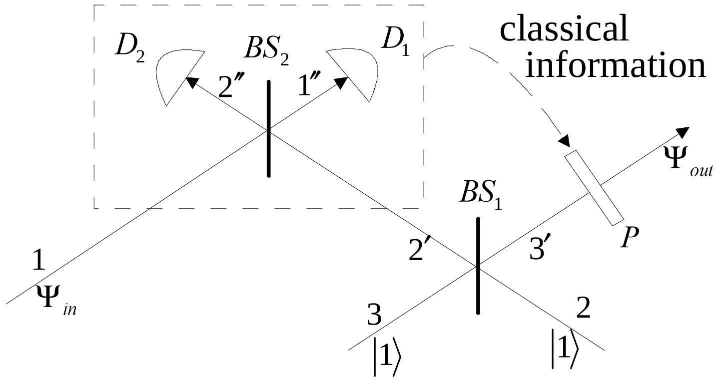
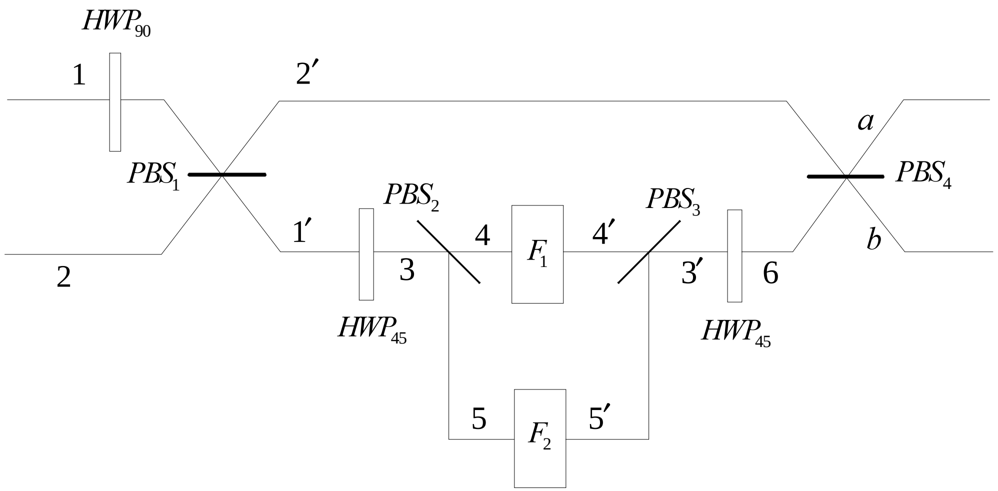
Figure Captions

Figure 1. The schematic shows the implementation of the single-mode quantum filter. BS_i(i=1,2) denotes the symmetric beam splitters and D_i(i=1,2,3) are photon number detectors. The $\pi/2$ phase shifter is denoted by P.
Figure 2. This scheme shows the implemention of the projective measurement (1). The half-wave plates HWP_{45} and HWP_{90} rotate the horizontal and vertical polarization by $\pi/4$ and $\pi/2$. The polarization beam splitters (PBS_i(i=1,2,3,4)) transmit H-photons and reflect V-Photons. F_i (i=1,2) denotes the single mode quantum filter, like it is shown in Fig.1.



classical
information
D_2
BS_2
D_1
$2''$
$1''$
Ψ_{out}
BS_1
P
1
$2'$
$3'$
Ψ_{in}
3
2
$|1\rangle$
$|1\rangle$



HWP_{90}
1
2′
2
PBS_1
a
PBS_4
b
1′
PBS_2
4
F_1
4′
PBS_3
3
3′
6
HWP_{45}
HWP_{45}
5
F_2
5′